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| **Rank of Matrix**  Let  be a non-zero matrix. A positive integer  is called the rank of the matrix  if   1. There exists at least one minor of order of  which is non-zero, and 2. Every minor of order greater than  is zero.   The rank of matrix  is denoted by. The rank in echelon form is equal to the number of non-zero rows of the matrix.  **Methods to find rank of a matrix**   1. Minor (determinant) method 2. Echelon form 3. Normal Form |
| **System of linear equations**  **Definitions, Formulae and Theory**  A linear system of m equations in n unknowns  is a set of equations of the form    This is called system of Linear equations. It is called linear because each variable  is of first degree only.  The given system can also be written in the matrix form as    i.e.  where  is the matrix of coefficients,  is the matrix of unknown variables and  is the matrix of constants. If, then the system is homogeneous. If, then the system is non-homogeneous.  The values of unknowns form a solution set of the given system. It satisfies all the equations of the system.  **Augmented Matrix**  Note that the matrix  is called the augmented matrix. It is represented as.  **Consistent**  If a system has solution then it is called consistent and vice-e-versa.  If a system has no solution then it is called inconsistent and vice-e-versa.    **To test consistency of the system of Linear equations**  **Method 1:**  If in the system there are n unknowns and n equations, then the matrix of coefficients i.e.  is a square matrix.  Consider    Applicable only ifand for square matrix.  **Method 2:**  This method can be used, if in the system there are n unknowns and m equations.  **For Non-Homogeneous Linear Equations:**  i) Write down the given system in the matrix form i.e. .  ii) Reduce the matrix  to row echelon form and do same transformation on matrix  also.  iii) As the matrix is in row echelon form, we can determine rank of matrix  and rank of augmented matrix.  iv) If Rank  < Rank , the system is inconsistent i.e. it has no solution.  v) If Rank = Rank , the system is consistent i.e. it has a solution.  Case a: If rank of  is equal to the number of unknowns, the system has unique solution.  Case b: If rank of  is less than the number of unknowns, the system has infinitely many solutions. Here, some unknowns are assigned arbitrary values called parameters. Number of parameters = (number of unknowns – rank of).  **For Homogeneous Linear Equations:**  i) Write down the given system in the matrix form i.e. .  ii) Reduce the matrix  to row echelon form.  iii) As the matrix is in row echelon form, we can determine rank of matrix.  iv) If Rank = number of unknowns, then the only possible solution is zero solution or trivial solution.  v) If Rank  < number of unknowns, then the system has non-trivial solution. Here, some unknowns are assigned arbitrary values called parameters. Number of parameters = (number of unknowns – rank of). |
| **LU Decomposition**  L U decomposition of a matrix is the factorization of a given square matrix into two triangular matrices, one upper triangular matrix and one lower triangular matrix, such that the product of these two matrices gives the original matrix. It was introduced by Alan Turing in 1948, who also created the turing machine.      **Using LU decomposition to solve systems of equations**  Once a matrix A has been decomposed into lower and upper triangular parts it is possible to obtain the solution to AX = B in a direct way. The procedure can be summarized as follows  • Given A, find L and U so that A = LU. Hence LUX = B.  • Let Y = UX so that LY = B. Solve this triangular system for Y .  • Finally solve the triangular system UX = Y for X |
| **Gaussian Elimination**  Gaussian elimination, our main method for finding the solution of a given system of linear equations, consists of using the row operations to transform a given system into an equivalent system whose solution can be easily obtained.  It essentially consists of two parts:   1. Part A. (Forward Elimination) Step-by-step reduction of the system yielding either a degenerate equation with no solution (which indicates the system has no solution) or an equivalent simpler system in triangular or echelon form.  * The first equation should have a leading coefficient of 1 . Interchange rows or multiply by a constant, if necessary. * Use row operations to obtain zeros down the first column below the first entry of 1 . * Use row operations to obtain a 1 in row 2, column 2. * Use row operations to obtain zeros down column 2, below the entry of 1. * Use row operations to obtain a 1 in row 3, column 3. * Continue this process for all rows until there is a \(1 in every entry down the main diagonal and there are only zeros below. * If any rows contain all zeros, place them at the bottom.  1. Part B. (Backward Elimination) Step-by-step back-substitution to find the solution of the simpler system. |
| **Vectors and Linear Combination**  **System of equations**    Can be rewritten as the following vector equation form        **Linear combination of vector**  A vector *v* is said to be linear combination of *u1,u2…..uk* if    **Theorem:** A system AX = B of linear equations has a solution if and only if B is a linear combination of the columns of the coefficient matrix A.  Thus, the answer to the problem of expressing a given vector v in Kn as a linear combination of vectors u1; u2; . . . ; um in Kn reduces to solving a system of linear equations. |